Halsted's book is lucidly written, the translation is at once faithful and smooth."

We congratulate Professor Halsted on the well-merited success that his book has attained. Sooner or later it will no doubt exercise a profound influence on the teaching of geometry in our country.

A METHOD FOR THE SOLUTION OF SIMULTANEOUS QUADRATIC EQUATIONS OF THE SYMMETRIC TYPE.

By B. E. MITCHELL, Nashville, Tennessee.

The following method is suggested but not developed in Fine's *College Algebra.*

The most general quadratic of the symmetric type between two variables is $a(x^2+y^2)+2bxy+c(x+y)+d=0$.

Let x and y be the roots of $u^2+pu+q=0$. Then

$$u=\begin{cases} x=\frac{1}{2}[-p\pm\sqrt{(p^2-4q)}] \\ y=\frac{1}{2}[-p\mp\sqrt{(p^2-4q)}] \end{cases}\ldots(\mathrm{I}).$$

The double sign is used in each because they are the roots of a symmetric equation.

From the theory of quadratic equations we have

$$x+y=-p\ldots(1),$$

and $$xy=q\ldots(2),$$

from which we calculate $$x^2+y^2=p^2-2q\ldots(3).$$

These are the three symmetric forms that occur in the general equation above, and are the only ones of the second degree.

Let us solve two simultaneous equations by this method.

$$\begin{cases} a_1(x^2+y^2)+2b_1xy+c_1(x+y)+d_1=0\ldots(1). \\ a_2(x^2+y^2)+2b_2xy+c_2(x+y)+d_2=0\ldots(2). \end{cases}$$

Then

$$\begin{cases} a_1p^2-2(a_1-b_1)q-c_1p+d_1=0\ldots(3). \\ a_2p^2-2(a_2-b_2)q-c_2p+d_2=0\ldots(4). \end{cases}$$

Each of the equations (3) and (4) is linear in q. Solving them for q, we have

$$2q=\frac{a_1p^2-c_1p+d_1}{a_1-b_1}=\frac{a_2p^2-c_2p+d_2}{a_2-b_2}\ldots(5).$$

Considering equation (5) we have the following:

The coefficient of p^2 is $a_2b_1-a_1b_2=-\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$ which we call A.

The coefficient of p is $a_1c_1-a_2c_2-b_1c_2+b_2c_1=-\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 0 \end{vmatrix}$ which we call B.

The constant term is $a_2d_1-b_2d_1-a_1d_2+b_1d_2=\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ d_1 & d_2 & 0 \end{vmatrix}$ which we call C.

Hence we have $Ap^2+Bp+C=0$ yielding generally two values for p, and from (5) we may calculate the two corresponding values of q. We have then the two equations in u:

$$u^2+p_1u+q_1=0,$$
$$u^2+p_2u+q_2=0,$$

which give four pairs of values for x and y when substituted in (I).

In the application of the method to numerical problems it will usually be found easier to use it as a process rather than the above equations as formulae.

Let us apply the method to three numerical problems taken from Fine's *College Algebra.*

(1) $\begin{cases} x+y=5. \\ xy+36=0. \end{cases}$

Hence $p=-5$ and $q=-36$, so $u^2-5u-36=0$, and $u=\begin{cases} x=9 \text{ and } -4. \\ y=-4 \text{ and } 9. \end{cases}$

(2) $\begin{cases} x^2+xy+y^2=21. \\ x+\sqrt{(xy)}+y=7. \end{cases}$ $\begin{cases} p^2-2q+q=21. \\ -p+\sqrt{q}=7. \end{cases}$ $\begin{cases} p^2-q=21. \\ p^2-q+14p=-49. \end{cases}$

$p=-5,\ q=4.\quad u^2-5u+4=0.\quad (u-4)(u-1)=0.$

$u=\begin{cases} x=1 \text{ and } 4. \\ y=4 \text{ and } 1. \end{cases}$

(3) $\begin{cases} x^2+3xy+y^2+2x+2y=8. \\ 2x^2+2y^2+3x+3y=14. \end{cases}$

Then $p^2-2p+q=8$, and $2p^2-3p-2q=14$.

Eliminating q by addition, $4p^2-7p-30=0$, $p=-2$ and $\frac{15}{4}$, then $q=-0$ and $\frac{23}{16}$. So $u^2-2u=0\ldots$(a), and $u^2+\frac{15}{4}u+\frac{23}{16}=0\ldots$(b).

From (a), $u=\begin{cases} x=0 \text{ and } 2. \\ y=2 \text{ and } 0. \end{cases}$

From (b), $u=\begin{cases} x=\frac{1}{2}\left[-\frac{15}{4}\pm\sqrt{\left(\frac{133}{16}\right)}\right]. \\ y=\frac{1}{8}\left[-15\mp\sqrt{(133)}\right]. \end{cases}$

This method may also be used where the equations are symmetric with respect to x and $-y$. In this case we have

$$x-y=-p,\ xy=-q, \text{ and } x^2+y^2=p^2-2q.$$

By calculating other symmetric functions the method can be used successfully in solving many equations of higher degree than the second.

SOME CONSTRUCTIONS LEADING TO CONICS.

By F. H. HODGE, Franklin College, Indiana.

Among the courses that find a place in collegiate mathematics one is usually given which involves the treatment of plane curves. Certain loci are described in such a course and their equations are derived and the properties indicated by the equations are discussed. It seems desirable to have a stock of plane loci which can be simply described, and the derivation of whose equations does not present great difficulty, but which are not treated in current text books. Such loci could be assigned to pupils as original exercises, and they would serve to stimulate the capacity for independent thought.

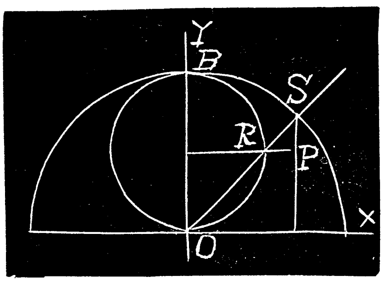

Fig. 1.

The four constructions which follow lead to conics. These are so simple and so directly analogous to a well known construction for the ellipse that it is scarcely possible that they are new though they are not found in current texts. The first two are described in detail, and the others merely suggested.

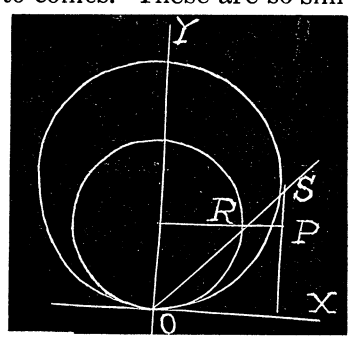

Fig. 2.

(1) Given two circles tangent internally at B and having the radius of the larger equal to the diameter of the smaller. Take the center of the larger circle as origin and the tangent to the smaller circle at that point as the x-axis, the y-axis being perpendicular to the x-axis. Draw a secant line through the origin, meeting the smaller circle at R and the larger circle at S. Through R draw a line parallel to the x-axis, and through S draw a line parallel to the y-axis. Call the point P in which these two lines meet. Required to find the locus of P as the secant line revolves about the origin as an axis. See Fig. 1.